Curvature Squared Action in $\mathcal{N} = 2$ Supergravity in Four Dimensions

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Motivation

- In low curvature limit, the entropy of black hole is given by Bekenstein - Hawking Area Law

$$S_{BH} = \frac{A}{4} \tag{1}$$

- From statistical point of view, the entropy of black hole is

$$S_{stat} = no.\ of\ microstates\ of\ black\ hole$$

- In the large black hole limit

$$S_{BH}(Q) = S_{stat}(Q)$$

where $S_{BH}(Q)$ denotes the Bekenstein-Hawking entropy of an extremal black hole carrying a given set of charges labelled by Q, and $S_{stat}(Q)$ is statistical entropy in the string theory carrying the same set of charges.

- For the finite size black hole, higher derivative terms modifies the horizon geometry hence, black hole entropy.
- Does the agreement between $S_{BH}(Q)$ and $S_{stat}(Q)$ continue to hold even after taking into account the effects of higher derivative corrections on the black hole side, and deviation from the Cardy formula on the statistical side?
- It has been realized that the entropy of black hole can't be completely determined by using area law if we include higher derivative terms.
- Entropy function formalism is used to calculate the entropy of a black hole in the presence of higher derivative terms in the action.

- **Connection with String Theory** : Higher derivative terms are important in establishing a connection between microscopic quantities computed in the string theory description, and macroscopic quantities computed in the effective supergravity description.

Outline

Gauge Equivalence Program

1 . Introduce the extra symmetry (algebra and gauge multiplet),

2 . Choose compensators,

3 . Construct invariant action,

4 . Choose gauge fixings for redundant symmetries,

5 . Rewrite action and transformations.

Schematic Representation

Conformal Gravity

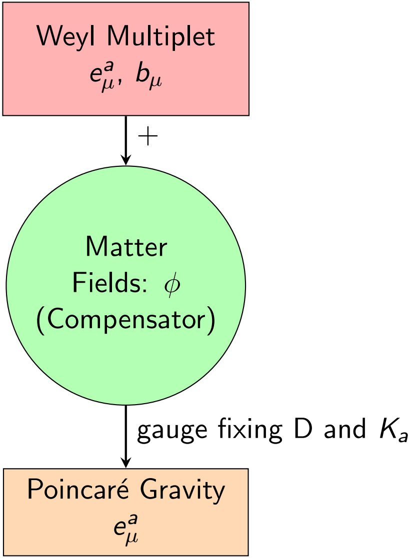
Independent fields: e_μ^a, b_μ

No. of d.o.f $= 5$

Poincaré Gravity

Independent fields: e_μ^a

No. of d.o.f $= 6$



Conformal Algebra and Gauge Fields

A general global conformal transformation is given as follows

$$\delta_C \Phi^i = \xi^\mu(x)\partial_\mu \Phi^i - \frac{1}{2}\Lambda_M^{\mu\nu}(x) m_{\mu\nu}\Phi^j + \Lambda_D(x) k_D^i(\Phi) + \lambda_K^\mu k_\mu^i(\Phi) \quad (2)$$

where

$$\begin{aligned} \xi^\mu(x) &= a^\mu + \lambda^{\mu\nu}x_\nu + x^\mu\lambda_D + \left(x^2\lambda_K^\mu - 2x^\mu x\cdot\lambda_K\right) \\ \Lambda_M^{\mu\nu}(x) &= \lambda^{\mu\nu} - 4x^{[\mu}\lambda_K^{\nu]} \\ \Lambda_D(x) &= \lambda_D - 2x\cdot\lambda_K \end{aligned}$$

Commutator algebra of the conformal group

$$\left[M_{[\mu\nu]}, M_{[\rho\sigma]}\right] = 2\eta_{\mu[\rho}M_{[\sigma]\nu]} - 2\eta_{\nu[\rho}M_{[\sigma]\mu]}, \quad \left[P_\mu, M_{[\nu\rho]}\right] = 2\eta_{\mu[\nu}P_{\rho]}$$

$$\left[K_\mu, M_{[\nu\rho]}\right] = 2\eta_{\mu[\nu}K_{\rho]}, \quad [P_\mu, K_\nu] = 2\left(\eta_{\mu\nu}D + M_{[\mu\nu]}\right)$$

$$[D, P_\mu] = P_\mu, \quad [D, K_\mu] = -K_\mu$$

Local Conformal Transformation

$$P_a \longrightarrow e_\mu^a, \quad M_{ab} \longrightarrow \omega_\mu^{ab}, \quad D \longrightarrow b_\mu, \quad K_a \longrightarrow f_\mu^a \tag{3}$$

$$a_\mu \longrightarrow a_\mu(x), \quad \lambda_{\mu\nu} \longrightarrow \lambda_{\mu\nu}(x), \quad \lambda_D \longrightarrow \lambda_\mu(x), \lambda_K^\mu \longrightarrow \lambda_K^\mu(x) \tag{4}$$

- $\xi^\mu(x)$ becomes an arbitrary function of spacetime and the first term in Eq(2) describes the transformation of Φ_i under a *g.c.t.*
- Conformal transformation $\longrightarrow$ Covariant *g.c.t.* + Standard gauge transformations.

▸ Details

$$\delta_{cgct} = \delta_{gct} - \delta_I(\xi^\mu h_\mu^I) \tag{5}$$

- The gauge fields transform under the standard conformal transformations as follows

$$\begin{aligned}
\delta e_\mu^a &= -\lambda^{ab} e_{\mu b} - \lambda_D e_\mu^a \\
\delta \omega_\mu^{ab} &= \partial_\mu \lambda^{ab} + 2\omega_{\mu c}^{[a} \lambda^{b]c} - 4\lambda_K^{[a} e_\mu^{b]} \\
\delta b_\mu &= \partial_\mu \lambda_D + 2\lambda_K^a e_{\mu a} \\
\delta f_\mu^a &= \partial_\mu \lambda_K^a - b_\mu \lambda_K^a + \omega_\mu^{ab} \lambda_{Kb} - \lambda^{ab} f_{\mu b} + \lambda_D f_\mu^a
\end{aligned}$$

Gauge Theory of Gravity

For fields to transform properly under covariant general coordinate transformations we need to impose constraints:

$$R_{\mu\nu}(P^a) = 0, \qquad e_b^\nu R_{\mu\nu}(M^{ab}) = 0.$$

Dependent gauge fields are given as:

$$\begin{aligned}
\omega_\mu^{ab} &= 2e^{\nu[a}\partial_{[\mu} e_{\nu]}{}^{b]} - e^{\nu[a} e^{\ b]\sigma} e_{\mu c}\partial_\nu e_\sigma^c + 2e_\mu{}^{[a} e^{\ b]\nu} b_\nu \\
f_\mu^a e_a^\mu &= -\frac{R}{4(D-1)}
\end{aligned}$$

Conformal Gravity Action

$$S_c = -\tfrac{1}{2}\int d^D x \; e\phi\Box^c\phi$$

where

$$\delta\phi = \lambda_D\phi \tag{6}$$

$$\Box^c\phi = e^{\mu a}(\partial_\mu\mathcal{D}_a\phi - 2b_\mu\mathcal{D}_a\phi + \omega_\mu^{ab}\mathcal{D}_b\phi + 2f_{\mu a}\phi)$$

Gauge fixing:

$$S_c = -\tfrac{1}{2}\int d^D x \; e\phi\Box^c\phi \quad \xrightarrow[\text{K-fixing}]{b_\mu\ = 0} \quad S_c' = \tfrac{1}{2}\int d^D x \; e\left(\partial_\mu\phi\partial^\mu\phi + \frac{(D-2)}{4(D-1)}R\phi^2\right)$$

Gauge Equivalent:

$$S_c' \quad \xrightarrow[\text{Breaks dilatations}]{\phi\ =\ \sqrt{\frac{4(D-1)}{(D-2)}}} \quad S_H = \tfrac{1}{2}\int d^D x \; eR$$

Superconformal Algebra

The superconformal transformations and their associated gauge fields are

Generator T	P_a	M_{ab}	D	K_a	Q^i	S^i	$V^i{}_j$	A
Gauge fields	e_μ^a	ω_μ^{ab}	b_μ	f_μ^a	$\psi_\mu{}^i$	$\phi_\mu{}^i$	$\mathcal{V}_\mu^i{}_j$	A_μ
Parameter	ξ^a	ε^{ab}	Λ_D	Λ_K^a	ε^i	η^i	$\Lambda^i{}_j$	Λ_A

$(1 \leq i, j \leq 2)$ -fundamental rep. of the SU(2) of R-symmetry,
(a,b $=$ 0......4) - tangent spacetime indices
$(\mu, \nu =$ 0....4) - curved or world indices.

Conformal Supergravity

To construct off-shell representation of $\mathcal{N} = 2$ conformal supergravity, we have to impose constraints on the conformal curvatures.

$$R(P)^{a}_{\mu\nu} = 0, \qquad e^{\nu}{}_{b} R(M)^{ab}_{\mu\nu} - i\tilde{R}(A)^{a}_{\mu} = 0 \qquad \gamma^{\mu} R(Q)^{i}_{\mu\nu} = 0. \tag{7}$$

Dependent bosonic fields: ω^{ab}_{μ}, f^{a}_{μ}

Dependent fermionic field: $\phi_{\mu}{}^{i}$

Gauge fields	e^{a}_{μ}	b_{μ}	ψ^{i}_{μ}	$\mathcal{V}^{i}_{\mu\ j}$

There are 17(bosonic) + 16(fermionic) off-shell degrees of freedom. Its necessary to add new fields to form super-multiplet.

Weyl Multiplet

All independent gauge fields together with the matter fields form a superconformal multiplet, called the **Weyl Multiplet**.

Gauge fields	e_μ^a	b_μ	ψ_μ^i	$\mathcal{V}_\mu{}^i{}_j$

Matter Fields

Dilaton Weyl Multiplet

$$W_\mu, \tilde{W}_\mu : \text{ gauge vectors (3+3)},$$
$$B_{\mu\nu} : \text{ Two-form gauge field (3)},$$
$$X : \text{ Complex scalar field (2)},$$
$$\Omega^i : \text{ Majorana spinors (8)}.$$

Make A_μ gauge field dependent on the matter fields.

There are 24 + 24 off-shell degrees of freedom.

Dilaton Weyl Multiplet in D = 4

$$
\begin{aligned}
\delta e_\mu{}^a &= \bar{\epsilon}^i \gamma^a \psi_{\mu i} + \text{h.c.} - \Lambda_D e_\mu{}^a + \Lambda_M^{ab} e_{\mu b}, \\
\delta \psi_\mu{}^i &= 2\mathcal{D}_\mu \epsilon^i - \frac{1}{8}\varepsilon^{ij}\gamma\cdot(\mathcal{F}^- + i\mathcal{G}^-)\gamma_\mu \epsilon_j - \gamma_\mu \eta^i \\
\delta b_\mu &= \frac{1}{2}\bar{\epsilon}^i \phi_{\mu i} - \frac{1}{4}\bar{\epsilon}^i X^{-1}\gamma_\mu \not{D}\Omega_i - \frac{1}{2}\bar{\eta}^i \psi_{\mu i} + \text{h.c.} + \Lambda_K^a e_{\mu a} + \partial_\mu \Lambda_D \\
\delta \mathcal{V}_\mu{}^i{}_j &= 2\bar{\epsilon}_j \phi_\mu^i - \bar{X}^{-1}\bar{\epsilon}_j \gamma_\mu \not{D}\Omega^i + 2\bar{\eta}_j \psi_\mu^i - (\text{h.c.};\ \text{traceless}) \\
\delta X &= \bar{\epsilon}^i \Omega_i + (\Lambda_D - i\Lambda_A)\, X \\
\delta \Omega_i &= 2\,\not{D} X \epsilon_i + \frac{1}{4}\epsilon_{ij}\gamma\cdot\mathcal{F}\epsilon^j - \frac{i}{4}\epsilon_{ij}\gamma\cdot\mathcal{G}^-\epsilon^j + 2X\eta_i \\
\delta W_\mu &= \varepsilon^{ij}\bar{\epsilon}_i \gamma_\mu \Omega_j + 2\varepsilon_{ij}\bar{X}\bar{\epsilon}^i \psi_\mu^j + \text{h.c.} + \partial_\mu \lambda \\
\delta \tilde{W}_\mu &= i\varepsilon^{ij}\bar{\epsilon}_i \gamma_\mu \Omega_j - 2i\varepsilon_{ij}\bar{X}\bar{\epsilon}^i \psi_\mu^j + \text{h.c.} + \partial_\mu \tilde{\lambda} \\
\delta B_{\mu\nu} &= \tfrac{1}{2} W_{[\mu}\delta_Q W_{\nu]} + \tfrac{1}{2}\tilde{W}_{[\mu}\delta_Q \tilde{W}_{\nu]} + \bar{X}\bar{\epsilon}^i \gamma_{\mu\nu}\Omega_i + 2\, X\bar{X}\bar{\epsilon}^i \gamma_{[\mu}\psi_{\nu]\, i} + h.c
\end{aligned}
$$

▸ Details

Butter, Daniel, et al. "N= 2 dilaton Weyl multiplet in 4D supergravity." Journal of High Energy Physics 2018.3 (2018): 154.

Tensor Multiplet

$$
\begin{aligned}
\delta L_{ij} &= 2\bar{\epsilon}_{(i}\phi_{j)} + 2\varepsilon_{ik}\varepsilon_{jl}\bar{\epsilon}^{(k}\phi^{l)} \ , \\
\delta\phi^{i} &= \not{D}L^{ij}\epsilon_j + \varepsilon^{ij}\not{E}\ \epsilon_j - G\epsilon^i + 2L^{ij}\eta_j \ , \\
\delta G &= -2\bar{\epsilon}_i\Gamma^i - \bar{\epsilon}_i(6L^{ij}\chi_j + \frac{1}{4}\gamma.T^{+}_{jk}\ \phi_l\ \varepsilon^{ij}\varepsilon^{kl}) + 2\bar{\eta}_i\ \phi^i \ , \\
\delta E_{\mu\nu} &= i\bar{\epsilon}^i\gamma_{\mu\nu}\phi^j\varepsilon_{ij} + 2\ iL_{ij}\varepsilon^{jk}\bar{\epsilon}^i\gamma_{[\mu}\psi_{\nu]k} - i\bar{\epsilon}^i\gamma_{\mu\nu}\phi^j\varepsilon_{ij} \\
&\quad -2\ iL_{ij}\varepsilon^{jk}\bar{\epsilon}^i\gamma_{[\mu}\psi_{\nu]k}.
\end{aligned}
\tag{8}
$$

$$\Gamma^i = \not{D}\phi^i - 2L^{ij}\chi_j$$

There are 8 + 8 off-shell degrees of freedom.

Van Proeyen, Antoine. "N= 2 supergravity in d= 4, 5, 6 and its matter couplings." extended version of lectures given during the semester "Supergravity, superstrings and M-theory" at Institut Henri Poincaré, Paris (2000).

Yang-Mills Multiplet

$$
\begin{aligned}
\delta A^I &= \bar{\epsilon}^i \lambda_i^I - g\alpha^J A^K f_{JK}^L \ , \\
\delta \lambda_i^I &= 2\not{D} A^I \epsilon_i + \frac{1}{2}\gamma.\hat{\mathcal{B}}^I \varepsilon_{ij}\epsilon^j + Y_{ij}^I \epsilon^j + 2A^I \eta_i - g\alpha^J \lambda_i^K f_{JK}^L \ , \\
\delta C_\mu^I &= \varepsilon^{ij}\bar{\epsilon}_i \gamma_\mu \lambda_j^I + 2\,\varepsilon_{ij}\bar{\epsilon}_i \psi_{\mu j} A^I + h.c. + \partial_\mu \alpha^I - g\alpha^J C_\mu^K f_{JK}^L \ , \\
\delta Y_{ij}^I &= 2\,\bar{\epsilon}_{(i}\Pi_{j)}^I + 2\,\varepsilon_{ik}\varepsilon_{jl}\bar{\epsilon}^{(k}\Pi^{l)I} - g\alpha^J Y_{ij}^K f_{JK}^L \ ,
\end{aligned}
$$

$$
\Pi_j = \not{D}\lambda_j - 2A\chi_j \tag{9}
$$

where I = 1,2,....n (n is the dimension of gauge group)

There are 8n + 8n off-shell degrees of freedom.

Details

Van Proeyen, Antoine. "N= 2 supergravity in d= 4, 5, 6 and its matter couplings." extended version of lectures given during the semester "Supergravity, superstrings and M-theory" at Institut Henri Poincaré, Paris (2000).

Linear Multiplet Composite Fields

$$\begin{aligned} L_{ij} &= Y_{ij}, \\ \phi_i &= \Pi_i = \not{D}\lambda_i - 2A\chi_i, \\ E^a &= 2D_b\left(\hat{\mathcal{B}}^{-ab} - \frac{1}{4}AT^{+ab}\right), \\ G^* &= -2\left(\Box^c A - AD + \bar{\chi}^k\lambda_k + \frac{1}{8}\hat{\mathcal{B}}.T^-\right) \end{aligned} \tag{10}$$

where superconformal d'Alembertian for A is given as

$$\begin{aligned} \Box^c A^I = D^a D_a A^I &= \mathcal{D}^a D_a A^I + 2f_a^a A^I - \frac{1}{2}\bar{\psi}_a^i D^a \lambda_i^I - \frac{1}{2}\bar{\lambda}_i \gamma^a \phi_a{}^i \\ &\quad + \frac{1}{8}X^{-1}A^I\bar{\psi}_a\gamma^a\not{D}\Omega_i - \frac{1}{16}\bar{\lambda}_{iI}\varepsilon^{ij}\bar{X}^{-1}\gamma.(\mathcal{F}^- + i\mathcal{G}^-)\gamma^a\psi_{aj} \\ &\quad - \frac{1}{2}g\varepsilon^{ij}\overline{\psi}_i \cdot \gamma\lambda_j^J X^K f_{JK}^I - \frac{1}{2}g\varepsilon_{ij}\overline{\psi}^i \cdot \gamma\lambda^{jJ}X^K f_{JK}^I \end{aligned} \tag{11}$$

Details

Yang-Mills Composite Fields

$$
\begin{aligned}
A &= G^* L^{-1} - \bar{\phi}_i \phi_j L^{ij} L^{-3}, \\
\lambda_i &= -2L^{-1}\, \not{D}\, \phi_i - (2L_{ij}\chi^j + \frac{1}{4}\varepsilon_{kl}\varepsilon_{ij}\, \phi^l\, \gamma\, .T^{-\,jk})\, L^{-1} \\
Y_{ij} &= 2[-L^{-1}\Box^c L_{ij} - DL^{-1}L_{ij} + D_a L_{ik} D^a L_{jl} L^{kl} L^{-3} + E^2 L_{ij} L^{-3} \\
&\quad + G^2 L_{ij} L^{-3} - E_\mu D^\mu L_{k(i} L_{j)}{}^{k} L^{-3} + h.c.] + fermions \\
\hat{\mathcal{B}}_{\mu\nu} &= \partial_{[\mu} L_{ik} \partial_{\nu]} L_{jl} L^{kl} \varepsilon^{ij} L^{-3} - 2\partial_{[\mu}(\, E_{\nu]} L^{-1} - \frac{1}{2}\mathcal{V}_{\nu]}{}^{k}{}_{i} L_{kj} \varepsilon^{ij} L^{-1}) \\
&\quad + h.c. + fermions
\end{aligned}
\tag{12}
$$

Density Formula

The density formula for the product of a vector multiplet and a linear multiplet is given as

$$\begin{aligned} e^{-1}\mathcal{L} &= AG + \bar{A}\bar{G} - \frac{1}{2}Y^{ij}L_{ij} + \frac{1}{4}e^{-1}\varepsilon^{\mu\nu\rho\sigma}E_{\mu\nu}B_{\rho\sigma} \\ &\quad +\bar{\phi}^i(\lambda_i + A\,\gamma^\mu\psi_{\mu\,i}) + \bar{\phi}_i(\lambda^i + \bar{A}\,\gamma^\mu\psi_\mu{}^i) \\ &\quad -\frac{1}{2}(\bar{\psi}_\mu^i\gamma^\mu\lambda^j + \bar{A}\,\bar{\psi}_\mu^i\gamma^{\mu\nu}\psi_\mu^j)L_{ij} + h.c. \end{aligned} \tag{13}$$

de Wit, Bernard, and Frank Saueressig. "Off-shell N= 2 tensor supermultiplets." Journal of High Energy Physics 2006, no. 09 (2006): 062.

Superconformal Actions

Superconformal Action for Yang-Mills Multiplet

$$\begin{aligned} e^{-1}\mathcal{L}_{YM} &= \frac{-a_{IJ}}{2}[Y^I_{ij}Y^{ijJ} + 4(\bar{A}^I\Box^c A^J - \bar{A}^I A^J D - \frac{1}{8}\bar{A}^I\hat{\mathcal{B}}^J.T^-) + h.c. \\ &+ 4e^{-1}\left(\hat{\mathcal{B}}^{\mu\nu\ I\ -} - \frac{1}{4}A^I T^{\mu\nu\ +}\right)B^J_{\mu\nu} + h.c. \\ &+(fermions)]. \end{aligned} \tag{14}$$

Superconformal Action for Linear Multiplet

$$\begin{aligned} e^{-1}\mathcal{L}_T &= DL + L^{-1}L^{ij}\Box^c L_{ij} + G^2L^{-1} + E^\mu E_\mu L^{-1} \\ &-E^\mu L^{-1}\mathcal{V}_\mu^k{}_i L_{kj}\varepsilon^{ij} - L^{ij}D_a L_{ik}D^a L_{jl}L^{kl}L^{-3} \\ &+\tilde{E}^{\mu\nu}\partial_\mu L_{ik}\partial_\nu L_{jl}L^{kl}\varepsilon^{ij}L^{-3} + (fermions). \end{aligned} \tag{15}$$

Gauge fixing and Decomposition Laws

Gauge Choices

$b_\mu = 0$ (K-gauge), $X = \bar{X}$ (A-gauge), $X = \bar{X} = 1$ (D-gauge),
$\Omega_i = 0$ (S-gauge) $L_{ij} = \delta_{ij}\frac{L}{\sqrt{2}}$ (SU(2) gauge $\to$ U(1)).

Decomposition Laws

$$\eta^i = \frac{1}{12}\gamma^a.\varepsilon_{abcd}\mathcal{H}^{bcd}\epsilon^i - \frac{1}{8}\varepsilon^{ij}\gamma.\mathcal{F}\epsilon_j - \frac{i}{8}\varepsilon^{ij}\gamma.\mathcal{G}^+\epsilon_j \qquad (16)$$

$$\Lambda_{K\mu} = -\frac{1}{2}\bar{\epsilon}^i\phi_{\mu i} + \frac{1}{2}\bar{\epsilon}^i\gamma_\mu\chi_i + \frac{1}{2}\bar{\eta}^i\psi_{\mu i} + h.c. \qquad (17)$$

$$\Lambda^{ij} = \frac{\sqrt{2}}{L}\left(\bar{\epsilon}^{(i}\phi^{j)} + \varepsilon^{ik}\varepsilon^{jl}\bar{\epsilon}_{(k}\phi_{l)} - \frac{1}{2}\delta^{ij}\,\bar{\epsilon}^{\,m}\phi^n\delta_{mn} + h.c.\right) \qquad (18)$$

Super-Poincaré Actions

SuperPoincaré Action for Yang-Mills Multiplet

$$
\begin{aligned}
e^{-1}\mathcal{L}_{YM} \;=\;& -\frac{1}{2}Y^I_{ij}Y^{ij}_I + 2\,(\,D'^a\bar{A}_I D'_a A^I + 2iA^I A^a D'_a\bar{A}_I - 2i\bar{A}^I A^a D'_a A_I \\
& -\frac{1}{4}\bar{A}^I B^{ab}_I . T^-_{ab} + \frac{1}{16}\bar{A}^I\bar{A}_I . T^-_{ab}T^{ab-}\,) + h.c. \\
& - e^{-1}\left(B_{\mu\nu}{}^I B^{\mu\nu}_I + \frac{1}{4}A^I B^{\mu\nu}_I T^+_{\mu\nu} + \frac{1}{4}\bar{A}^I B^{\mu\nu}_I T^-_{\mu\nu}\right) + \; h.c. \\
& + \;(\mathit{fermions}). \qquad (19)
\end{aligned}
$$

SuperPoincaré Action for Linear Multiplet

$$
\begin{aligned}
e^{-1}\mathcal{L}_{TR} \;=\;& \frac{1}{2}LR - \frac{1}{2}L^{-1}\partial_\mu L\partial^\mu L + \frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{4}G^{\mu\nu}G_{\mu\nu} + G^2L^{-1} \\
& + \frac{L}{24}H^{\mu\nu\rho}H_{\mu\nu\rho} + L^{-1}E_\mu E^\mu - L\;V'^{\;ij}_{\mu}V'^{\;\mu}_{ij} + \mathit{fermions} \qquad (20)
\end{aligned}
$$

New Transformation Rules

The Poincare supergravity is invariant under following supersymmetric transformations :

$$\begin{aligned}
\delta e_\mu{}^a &= \bar{\epsilon}^i \gamma^a \psi_{\mu\, i} + h.c. + \Lambda_M^{ab} e_{\mu b}, \\
\delta \psi_\mu{}^i &= 2 D'_\mu(\omega_+)\epsilon^i - \frac{1}{2}\varepsilon^{ij}\gamma^\rho(\widehat{\mathcal{F}}_{\rho\mu} + i\widehat{\mathcal{G}}_{\rho\mu})\epsilon_j, \\
\delta W_\mu &= 2\varepsilon_{ij}\bar{\epsilon}^i \psi_\mu^j + h.c. + \partial_\mu \lambda\ , \\
\delta \tilde{W}_\mu &= -2\ i\ \varepsilon_{ij}\bar{\epsilon}^i \psi_\mu^j + h.c. + \partial_\mu \tilde{\lambda}\ , \\
\delta B_{\mu\nu} &= \frac{1}{2} W_{[\mu}\delta_Q W_{\nu]} + \frac{1}{2}\tilde{W}_{[\mu}\delta_Q \tilde{W}_{\nu]} + 2\bar{\epsilon}^i \gamma_{[\mu}\psi_{\nu]i} + 2\bar{\epsilon}_i \gamma_{[\mu}\psi^i_{\nu]}.
\end{aligned} \tag{21}$$

Supercovariant Curvature

The Supercovariant Curvature under the Gauge as

$$\begin{aligned}
\widehat{\psi}_{\mu\nu}{}^{i} &= 2D'_{[\mu}(\omega_{+})\psi_{\nu]}{}^{i} - \frac{1}{2}\varepsilon^{ij}\gamma^{c}\mathcal{F}_{c[\mu}\psi_{\nu]\,j} - \frac{i}{2}\varepsilon^{ij}\gamma^{c}\mathcal{G}_{c[a}\psi_{b]\,j}, \\
\widehat{\mathcal{F}}_{\mu\nu} &= 2\partial_{[\mu}W_{\nu]} - \varepsilon_{ij}\bar{\psi}^{i}_{[\mu}\psi^{j}_{\nu]} - \varepsilon^{ij}\bar{\psi}_{[\mu i}\psi_{\nu]\,j}, \\
\widehat{\mathcal{G}}_{\mu\nu} &= 2\partial_{[\mu}\tilde{W}_{\nu]} + i\,\varepsilon_{ij}\bar{\psi}^{i}_{[\mu}\psi^{j}_{\nu]} - i\,\varepsilon^{ij}\bar{\psi}_{[\mu i}\psi_{\nu]\,j}, \\
\widehat{\mathcal{H}}_{\mu\nu\rho} &= 3\partial_{[\mu}B_{\nu\rho]} + \frac{3}{4}W_{[\mu}F_{\nu\rho]} + \frac{3}{4}\tilde{W}_{[\mu}G_{\nu\rho]} - 3\bar{\psi}_{[\mu}{}^{i}\gamma_{\nu}\psi_{\rho]\,i}.
\end{aligned} \qquad (22)$$

Transformation of Supercovariant Curvatures

Supersymmetric transformation of supercovariant curvatures are

$$
\begin{aligned}
\delta\widehat{\mathcal{K}}_{ab} &= 4\varepsilon_{ij}\bar{\epsilon}^i\hat{\psi}^j_{ab} + h.c.\,, \\
\delta\widehat{\psi}^i_{ab} &= -\frac{1}{2}\gamma^{cd}\widehat{R}_{cdab}(\omega_-)\epsilon^i + (\widehat{V}_{ab})^i{}_j\epsilon^j - \frac{\varepsilon^{ij}}{2}\not{D}'(\omega_-)\widehat{\mathcal{K}}_{ab}\epsilon_j \\
&\quad -\frac{1}{4}\widehat{\mathcal{F}}_{ab}\gamma.\widehat{\mathcal{F}}\epsilon^i - \frac{1}{4}\widehat{\mathcal{G}}_{ab}\gamma.\widehat{\mathcal{G}}\epsilon^i + \frac{i}{4}\eta^{cd}[\widehat{\mathcal{G}}_{ac}\widehat{\mathcal{F}}_{db} + \widehat{\mathcal{F}}_{ac}\widehat{\mathcal{G}}_{db}]\,, \\
\delta\widehat{\omega}_{-\mu ab} &= \bar{\epsilon}_i\gamma_\mu\widehat{\psi}^i_{ab} - \frac{1}{2}\varepsilon^{ij}\bar{\epsilon}_i\widehat{\mathcal{K}}_{ab}\psi_{\mu j}
\end{aligned}
\tag{23}
$$

where

$$
\begin{aligned}
\omega_{\mu\pm}^{ab} &= \omega_\mu{}^{ab} \pm \frac{1}{2}\widehat{\mathcal{H}}_\mu{}^{ab} \\
\delta\widehat{K}_{ab} &= \widehat{\mathcal{F}}_{ab} + i\widehat{\mathcal{G}}_{ab},
\end{aligned}
\tag{24}
$$

Yang-Mills Fields under the Gauge condition

$$
\begin{aligned}
\delta A^I &= \bar{\epsilon}^i \lambda_i^I \ , \\
\delta \lambda_i^I &= \frac{1}{2}\gamma.\mathcal{B}^{I-}\varepsilon_{ij}\epsilon^j + Y_{ij}^I \epsilon^j + 2\not{D}' A^I \epsilon_i - \frac{\varepsilon_{ij}}{4} A^I \gamma.(\widehat{\mathcal{F}} - i\widehat{\mathcal{G}})\epsilon^j - \frac{\varepsilon_{ij}}{4} \bar{A}^I \gamma.(\widehat{\mathcal{F}} + i\widehat{\mathcal{G}})\epsilon^j \\
&\quad + g A^J \bar{A}^K f_{JK}^I \varepsilon_{ij}\epsilon^j \ , \\
\delta C_\mu^I &= \varepsilon^{ij} \bar{\epsilon}_i \gamma_\mu \lambda_j^I + 2\, \varepsilon^{ij} \bar{\epsilon}_i \psi_{\mu j} A^I + h.c. \ ,
\end{aligned} \tag{25}
$$

where, $D'_\mu A^I = \partial_\mu A^I + g C_\mu^J A^K f_{JK}^I$

Map between Yang-Mills and Poincaré Multiplet

We observe that the transformation rules in Eq(25) and Eq(23) become identical upon making the following identifications:

$$\begin{pmatrix} A^I \\ \lambda_i^I \\ C_\mu^I \\ \dots \end{pmatrix} \leftrightarrow \begin{pmatrix} \frac{\widehat{\mathcal{K}}^{ab}}{4} \\ \varepsilon_{ij}\widehat{\psi}^{abj} \\ -\omega_{\mu-}^{ab} \\ \dots \end{pmatrix}$$

Using this observation we can now easily write down a supersymmetric Riemann tensor squared action by using the superPoincaré action formula for the Yang-Mills multiplet given in Eq(19).

Supersymmetrized Riemann Tensor Squared Action

We obtain the bosonic part of the supersymmetrized Riemann Tensor Squared Action(not complete)

Riemann Tensor Squared Action

$$
\begin{aligned}
e^{-1}\mathcal{L}_{Riem^2} \quad = \quad & -e^{-1}\left(R_{cdab}R^{abcd} + \frac{1}{4}R_{cdab}(\omega_-)(F^{ab}F^{cd} + G^{ab}G^{cd})\right) + \ h.c. \\
& + \frac{1}{8}(\ D'^{c}(\omega_-)F_{ab}D'_{c}(\omega_-)F^{ab} + D'^{c}(\omega_-)G_{ab}D'_{c}(\omega_-)G^{ab}) \\
& - \frac{1}{2}Y^{I}_{ij}Y^{ij}_{I} \\
& + \frac{1}{64}(F^{ab}F^{cd} + G^{ab}G^{cd})(F_{ab}F_{cd} + G_{ab}G_{cd}) + h.c. \\
& + \ (fermions).
\end{aligned} \tag{26}
$$

Future Work

- Using the Riemann tensor squared action we would like to calculate the entropy of the black hole in this theory, and we would like to see if it matches with the entropy in the string description.
- Apply localization technique and quantum entropy function formalism to calculate the quantum entropy of the black hole.

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Thank you

Examples

Proca Lagrangian describing a non-interacting massive vector field V_μ

$$\mathcal{L} = -\frac{1}{4}(\partial_\mu V_\nu - \partial_\nu V_\mu)^2 - \frac{1}{2}m^2 V_\mu^2$$

The presence of the mass term breaks the explicit gauge invariance of the kinetic term, $\delta V_\mu = \partial_\mu \lambda$. To obtain a locally gauge invariant Lagrangian via the field redefinition

$$V_\mu = W_\mu - m^{-1}\partial_\mu \phi \tag{27}$$

which is invariant under the combined local gauge transformations

$$\delta W_\mu = \partial_\mu \lambda \qquad \delta\phi = m\lambda \qquad \textit{with}\ \ \mathcal{D}_\mu \phi = \partial_\mu \phi - mW_\mu$$

Gauge theory of Superconformal Algebra

1- Assign gauge fields to each generator.

Generator T	P_a	M_{ab}	D	K_a	Q^i	S^i	$V^i{}_j$	A
Gauge fields $h_\mu(T)$	e_μ^a	ω_μ^{ab}	b_μ	f_μ^a	$\psi_\mu{}^i$	$\phi_\mu{}^i$	$\mathcal{V}_\mu{}^i{}_j$	A_μ
Parameter	ξ^a	ε^{ab}	Λ_D	Λ_K^a	ε^i	η^i	$\Lambda^i{}_j$	Λ_A

2-Transformations of gauge fields under local symmetry.

$$\delta(\epsilon)h_\mu{}^A = \partial_\mu\epsilon^A + \epsilon^C h_\mu{}^B f_{BC}{}^A \tag{28}$$

3- Covariant derivative is defined as:

$$\mathcal{D}_\mu \equiv \partial_\mu - \delta_A(h_\mu{}^A) \tag{29}$$

where $h_\mu{}^A$ means subtract the transformation of the gauge field, such that parameter is replaced by corresponding gauge field. Go Back

4-Modification in the definition of Covariant Derivative.

We relaize that we need to modify the definition of covariant derivative in presence of local translation. Because, in that case we can write covariant derivative of the scalar(without any internal symmetry) as

$$\mathcal{D}_\mu \phi = \partial_\mu \phi - e_\mu{}^a \partial_a \phi = 0 \tag{30}$$

Modified Covariant Derivative:

$$\mathcal{D}_\mu \equiv \partial_\mu - \delta_I(h_\mu{}^I) \tag{31}$$

where I denotes 'Standard gauge transformations'(All the gauge transformations except local translations).

4-Curvature or Field strength corresponding to gauge fields

$$[\mathcal{D}_\mu, \mathcal{D}_\nu] = -\delta R_{\mu\nu}$$

$$R_{\mu\nu}{}^A = 2\partial_{[\mu} h_{\nu]} + h_\nu{}^C h_\mu{}^B f_{BC}{}^A \tag{32}$$

$$\delta(\epsilon) R_{\mu\nu}{}^A = \epsilon^C R_{\mu\nu}{}^B f_{BC}{}^A \tag{33}$$

5-Gauge Theory for Spacetime Symmetries:

- $\omega_{\mu\nu}{}^{a}$ should not be an independent field in general relativity. It is a function of $e_{\mu}{}^{a}$.
- Translations will be replaced by general coordinate transformations (gct). These gct will have to be distinguished from the other transformations.
- General coordinate transformations on gauge fields acts as:

$$\delta_{gct}(\xi)h_{\mu}{}^{A} = \xi^{\nu}\partial_{\nu}h_{\mu}{}^{A} + (\partial_{\mu}\xi^{\nu})h_{\nu}{}^{A} \tag{34}$$

- Covariant general coordinate transformations is defined as

$$\delta_{cgct} = \delta_{gct} - \delta_{I}(\xi_{I}h_{\mu}{}^{I}) \tag{35}$$

◂ Go Back

Constraints

- To identify local translation as covariant general coordinate transformation we need to impose some constraints on the curvatures.

$$\delta_{cgct} e_\mu{}^a = \delta_P e_\mu{}^a - \xi^\nu R^a_{\mu\nu}(P) \tag{36}$$

- Constrainst

$$R(P)^a_{\mu\nu} = 0, \qquad e^\nu{}_b R(M)^{ab}_{\mu\nu} = 0 \qquad \gamma^\mu R(Q)^i_{\mu\nu} = 0. \tag{37}$$

- The constraints make the gauge fields $\omega_\mu{}^{ab}$, $f_\mu{}^a$ and ϕ^i dependent.

Counting of off-shell degrees of freedom

Off-shell degrees of freedom = number of field components - gauge transformation(symmetry)

Bosonic off-shell degree of freedom

Generator	Gauge fields	No. of field components	Symmetries	Off-shell DOF
P_a	e_μ^a	$4 \times 4 = 16$	4	12 - 6 - 4
M_{ab}	ω_μ^{ab}	$6 \times 4 = 24$	6	
D	b_μ	4	1	3
K_a	f_μ^a	$4 \times 4 = 16$	4	
$V^i{}_j$	$\mathcal{V}_\mu{}^i{}_j$	$3 \times 4 = 12$	3	9
A	$-iA_\mu$	4	1	3
Total		36	19	**17**

Fermionic off-shell degree of freedom

Q^i	$\psi_\mu{}^i$	$2 \times 4 \times 4 = 32$	$2 \times 4 = 8$	24 - 8
S^i	$\phi_\mu{}^i$	$2 \times 4 \times 4 = 32$	$2 \times 4 = 8$	
Total		32	16	**16**

Go Back

In the above equation, $\mathcal{F}_{\mu\nu}$ is the superconformally covariant field strength associated with the gauge field W_μ and is given as

$$\widehat{\mathcal{F}}_{\mu\nu} = 2\partial_{[\mu} W_{\nu]} - \varepsilon_{ij}\bar{\psi}_{[\mu}{}^{i}\Big(\gamma_{\nu]}\Omega^{j} + \psi_{\nu]}{}^{j}\bar{X}\Big) - \varepsilon^{ij}\bar{\psi}_{[\mu i}\Big(\gamma_{\nu]}\Omega_{j} + \psi_{\nu]j}X\Big) \, .$$

$$\widehat{\mathcal{G}}_{\mu\nu} = 2\partial_{[\mu} \tilde{W}_{\nu]} + i\varepsilon_{ij}\bar{\psi}_{[\mu}{}^{i}\Big(\gamma_{\nu]}\Omega^{j} + \psi_{\nu]}{}^{j}\bar{X}\Big) - i\varepsilon^{ij}\bar{\psi}_{[\mu i}\Big(\gamma_{\nu]}\Omega_{j} + \psi_{\nu]j}X\Big) \qquad (38)$$

Fundamental fields of standard weyl multiplet appears as composite fileds in dilaton weyl multiplet. The expressions for the composite fields are as follows:

$$T^{+ab} = 2\,X^{-1}\left(\mathcal{F}^{+ab} - i\mathcal{G}^{+ab}\right), \quad T^{-ab} = 2\,\bar{X}^{-1}\left(\mathcal{F}^{-ab} + i\mathcal{G}^{-ab}\right),$$

$$\chi_j = \frac{1}{2}X^{-1}\,\not{D}\Omega_j \, ,$$

$$D = \frac{1}{2}|X|^{-2}\left(X\Box^c\bar{X} + \frac{1}{2}\bar{\Omega}^k\,\not{D}\Omega_k + \frac{1}{4}\mathcal{F}\cdot\mathcal{F}^+ + \frac{1}{4}\mathcal{G}\cdot\mathcal{G}^+ + \text{h.c.}\right) \qquad (39)$$

where,

$$\begin{aligned}\Box^c X \quad &= \quad \mathcal{D}^a D_a X - \frac{1}{2}\bar{\psi}_a{}^{i} D^a\Omega_i + 2f_a^a X \\ &+\frac{1}{8}\bar{\psi}_a\gamma^a\not{D}\Omega_i + h.c. - \frac{1}{16}\bar{\Omega}_i\varepsilon^{ij}\bar{X}^{-1}\gamma.(\mathcal{F}^- + i\mathcal{G}^-)\gamma^a\psi_{aj} \\ &-\frac{1}{4}\bar{\Omega}_i\gamma^a\phi_a{}^{i} - \frac{1}{2}X^{-1}\bar{\Omega}_i\not{D}\Omega^i\end{aligned} \qquad (40)$$

Here we have introduced a superconformal field strength $\hat{\mathcal{B}}_{\mu\nu}$, defined by

$$\hat{\mathcal{B}}^{I-}_{\mu\nu} = \mathcal{B}^{I}_{\mu\nu} - \frac{1}{4}\bar{A}^{I}\ T^{-}_{\mu\nu} \tag{41}$$

where,

$$\begin{aligned}\mathcal{B}_{\mu\nu} &= 2\partial_{[\mu}C_{\nu]} - \bar{\psi}_{[\mu\ i}\gamma_{\nu]}\lambda_j\varepsilon^{ij} - \bar{\psi}_{[\mu}{}^{i}\gamma_{\nu]}\lambda^{j}\varepsilon_{ij} \\ &\quad -A\bar{\psi}_{\mu i}\psi_{\nu j}\varepsilon^{ij} - \bar{A}\,\bar{\psi}_{\mu}{}^{i}\psi_{\nu}{}^{j}\varepsilon_{ij},\end{aligned} \tag{42}$$

where the supercovariant derivative is defined as

$$\begin{aligned} D_\mu Y_{ij} &= (\partial_\mu - 2b_\mu)\ Y_{ij} - \mathcal{V}_\mu{}^{k}{}_{(i}Y_{j)k} - \bar{\psi}_{\mu(i}\not{D}\lambda_{j)} - \varepsilon_{ik}\varepsilon_{jl}\bar{\psi}^{(k}_{\mu}\not{D}\lambda^{l)}\ , \\ D_\mu\lambda_i &= (\partial_\mu - \frac{1}{4}\omega_\mu{}^{ab}\gamma_{ab} + \frac{i}{2}A_\mu - \frac{3}{2}b_\mu)\ \lambda_i - \frac{1}{2}\mathcal{V}_\mu{}^{j}{}_{i}\lambda_j \\ &\quad -\not{D}A\psi_{\mu\ i} - \frac{\varepsilon_{ij}}{4}\gamma.\hat{\mathcal{B}}\ \psi_\mu{}^{i} - \frac{1}{2}Y_{ij}\psi^{j}_{\mu} - A\phi_{\mu i} + g\ C^{J}_{\mu}\lambda^{K}_{i}f^{I}_{JK}\ , \\ D_\mu A &= (\partial_\mu - b_\mu + iA_\mu)A - \frac{1}{2}\bar{\psi}^{i}_{\mu}\lambda_i + g\ C^{J}_{\mu}A^{K}f^{I}_{JK} \end{aligned} \tag{43}$$

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Entropy Function Formalism

An algorithm for constructing an 'entropy function', – a function of the parameters labelling the near horizon background.

$$\mathcal{E}(\vec{u},\vec{v},\vec{e},\vec{q},\vec{p}) \equiv 2\pi\left(e_i q_i - f(\vec{u},\vec{v},\vec{e},\vec{p})\right) \tag{44}$$

where $f(\vec{u},\vec{v},\vec{e},\vec{p}) = \int d\theta d\phi \sqrt{-\det g}\,\mathcal{L}$.

Extremization of function $\mathcal{E}$ determines all the near horizon parameters

$$\frac{\partial \mathcal{E}}{\partial u_i} = 0 \;\;, \;\; \frac{\partial \mathcal{E}}{\partial v_i} = 0 \;\;, \;\; \frac{\partial \mathcal{E}}{\partial e_i} = 0.$$

This function at the extremum gives the entropy of the black hole

$$S_{BH} = \mathcal{E}(\vec{u},\vec{v},\vec{e},\vec{q},\vec{p})_{|near\ horizon\ background} \tag{45}$$